11.9 The Gas Laws—Boyle's Law, Gay-Lussac's Law, and the Combined Gas Law

Take a deep breath! Now do it again and think about what is happening with your rib cage, your diaphragm, and your lungs. Finally, take a third deep breath and think about how your body moves during breathing to draw oxygen-rich gas into your lungs and, in turn, exhale carbon dioxide-rich gas.

The process of breathing, which you undertake every few seconds, is a demonstration of another gas law. Your lungs are just like large elastic bags that can expand and contract. Your diaphragm is a muscle that extends across the bottom of your rib cage. When you inhale **(Figure 1(a))**, your diaphragm contracts while your rib cage expands. This allows your lungs to occupy more volume and, as a result, the gas in your lungs acquires a lower pressure. Air moves to the area of lower pressure and thus air enters your lungs. When you exhale **(Figure 1(b))**, your diaphragm rises and your rib cage contracts. This decreases the volume of your lungs and consequently creates a high pressure area. This causes the gas to move out of your lungs.

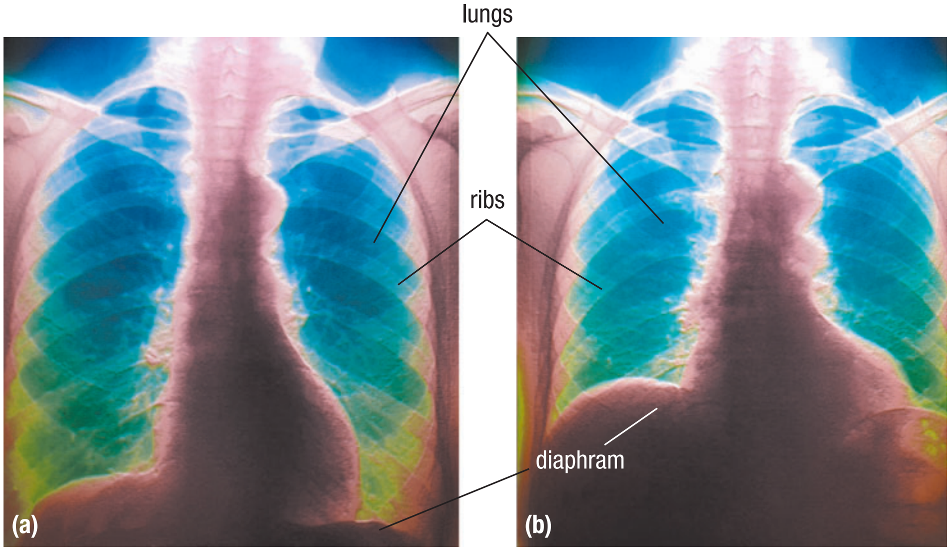

Figure 1 During (a) inhalation the volume of your lungs increases, whereas during (b) exhalation the volume decreases.

Control of the breathing process is of extreme importance in many different types of activities. Singers and wind or brass musicians need to have control of their breathing to achieve the appropriate tone, length of sound and pitch **(Figure 2)**. Competitive swimmers and runners need to monitor their breathing to maximize the delivery of oxygen to their bodies.

Figure 2 Playing a brass or woodwind instrument requires controlled breathing.

Boyle's Law

Boyle's law the statement that as the volume of a gas is decreased, the pressure of the gas increases proportionally, provided that the temperature and amount of gas remain constant; the volume and pressure of a gas are inversely proportional

To recap what happens during the breathing process, as the volume of a gas increases, the pressure of the gas decreases, as long as the temperature remains constant. The breathing process is an example of **Boyle's law**. This law, named for the British scientist Robert Boyle, states that as the volume of a gas is decreased, the pressure of the gas increases proportionally—provided that the temperature and amount of gas remain constant.

Mini Investigation

Modelling a Lung

Skills: Performing, Observing, Evaluating, Communicating

SKILLS HANDBOOK A2.1, 2.4

It is quite easy to create a model lung to demonstrate how Boyle's law is connected to the breathing process.

Equipment & Materials: straw; Styrofoam cup or similar container; 2 balloons; clear tape

1. Punch a hole the size of the straw in the bottom of the cup.
2. If the cup is not transparent, cut a small window in the side of the cup and seal the window with clear tape.
3. Place a balloon at the end of the straw.
4. Slide the other end of the straw through the hole in the bottom of the cup so that the balloon is inside the cup.
5. Cut about 1 cm off the top (sealed) end of the other balloon. Tie a knot in the neck of the remaining part. Stretch the balloon across the open end of the cup (**Figure 3**).
6. Pull down on the balloon cover (the diaphragm) and, through the window, observe what happens to the balloon inside the cup. Record your observations.

A. Summarize the effect of expanding the volume of the gas on the balloon inside. Explain, referring to the KMT. K/U

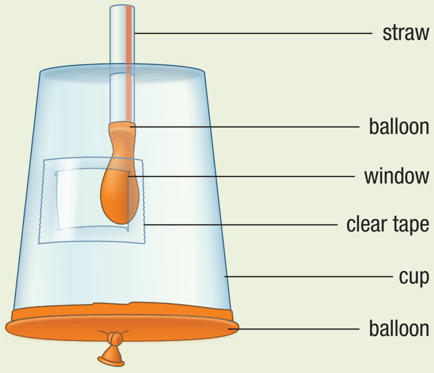

Figure 3

B. Connect and communicate this model to the body parts and processes used in breathing. K/U C

C. How accurate is this model? List some strengths and some weaknesses of this model. T/I

Volume–Pressure Relationships

SKILLS HANDBOOK A6. 4

We will use the data in **Table 1** to investigate the relationship between pressure and the volume of a gas. As usual, a mathematical relationship is easier to understand when plotted as a graph (**Figure 4**). In this case, the volume of the gas is the manipulated (independent) variable and the pressure of the gas is the responding (dependent) variable. Note that the temperature and the amount of gas remain constant throughout this investigation.

Table 1 Pressure versus Volume Data for a Closed Sample of Gas

Volume, V (L)	Pressure, P (kPa)	PV (L•kPa)	$1/P$ (kPa^{-1})
0.500	818	409	0.00122
1.00	411	411	0.00243
1.50	273	410	0.00366
2.00	205	410	0.00488
2.50	164	410	0.00610
3.00	137	411	0.00728
3.50	117	410	0.00850
4.00	101	404	0.00990

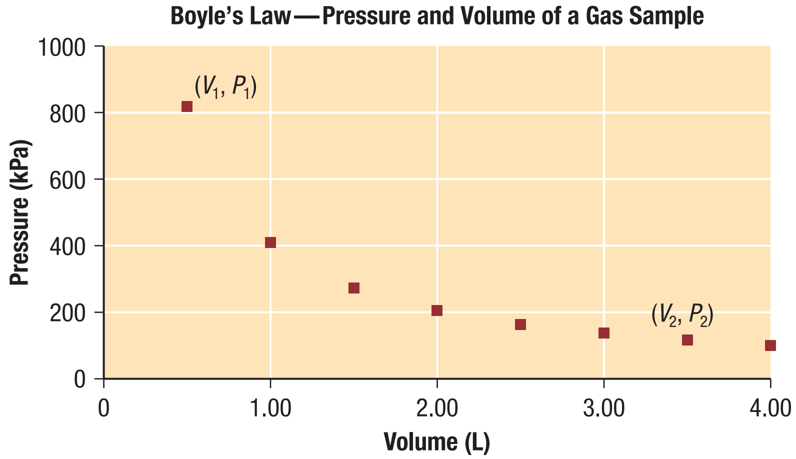

Figure 4 Pressure versus volume data for a closed sample of gas. Temperature and amount of gas have been kept constant.

The data in Figure 4 seem to show an inverse relationship. We can check this by plotting the inverse of pressure ($1/P$) against volume. If the pressure and volume of a gas are inversely related, then the graph of $1/P$ and V will produce a straight line. The graph in **Figure 5** on the next page confirms that there is indeed an inverse relationship between pressure and volume (at constant temperature and amount of gas).

Investigation 11.9.1

The Relationship between Volume and Pressure (p. 564)
In this controlled study you will manipulate one variable (volume) and measure the responding variable (pressure). You will control temperature and amount of gas.

LEARNING TIP

Inverse Relationship
When one variable is changed (increased or decreased) and the other responds in the opposite manner, the two variables are inversely related (or inversely proportional) to each other. As a real-world example, as the number of people buying tickets to a concert increases, the number of available seats decreases. These variables are inversely proportional.

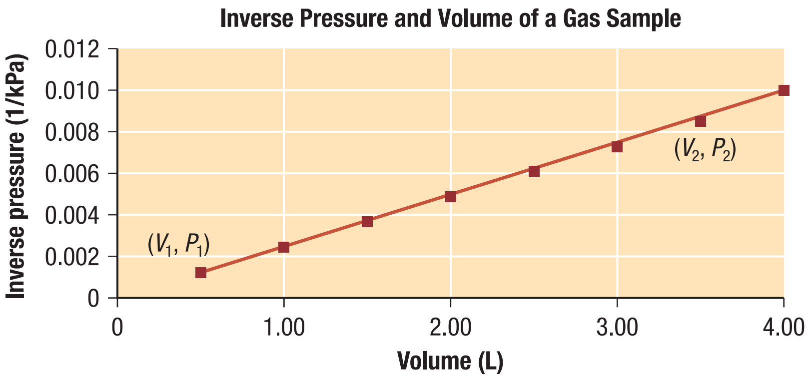

Figure 5 When we plot the inverse of pressure against volume (at constant temperature and amount of gas), the graph indicates a direct relationship.

The data in the *PV* column of Table 1 (page 555) show that the product of pressure and volume gives a constant value. Therefore, the product of any two points in Figure 4 should equal a constant, *k*. In this case, the constant is equal to 409 kPa·L, as follows:

$$P_1V_1 = k \qquad P_2V_2 = k$$

$$818\text{ kPa} \times 0.500\text{ L} = 409\text{ kPa·L} \qquad 117\text{ kPa} \times 3.50\text{ L} = 409\text{ kPa·L}$$

Since the two equations above both equal a constant, *k*, then, in general

$$P_1V_1 = P_2V_2$$

This relationship is the mathematical expression of Boyle's law.

Boyle's Law

$$P_1V_1 = P_2V_2\text{, or } PV = k \text{ (}k\text{ is a constant)}$$

The volume of a gas is inversely proportional to its pressure when the amount of gas and the temperature remain constant.

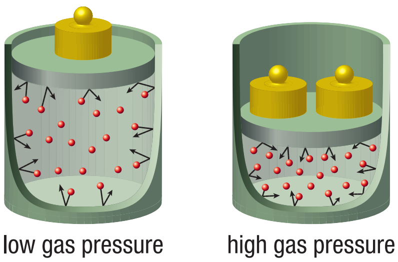

Figure 6 As the volume of the gas decreases, the pressure exerted by the gas increases.

It is important to picture what is happening to the gas molecules as we explore Boyle's law. Consider a fixed amount of a confined gas at a constant temperature. If we were to reduce the size of the container (decreasing the volume), the entities of gas would have less space in which to move. In a reduced space the entities would collide with each other and with the walls of the container more often. The pressure of the gas on the container would increase (**Figure 6**).

Mini Investigation

Cartesian Diver

Skills: Performing, Observing, Analyzing, Communicating — SKILLS HANDBOOK A2.4

A Cartesian diver is named for René Descartes, a famous scientist and mathematician (1592–1650). The "diver" in this investigation is a medicine dropper (**Figure 7**).

Equipment and Materials: empty water or pop bottle with cap; medicine dropper; water

1. Fill the pop bottle with tap water to within a few centimetres of the top.
2. Half-fill a glass medicine dropper with water.
3. Place the medicine dropper in the pop bottle. Adjust the volume of water in the medicine dropper until it floats upright. Cap the bottle securely.
4. Slowly squeeze the pop bottle. Observe the effect.

A. Explain why the diver initially floats. K/U T/I

B. What happened when you applied pressure to the pop bottle? Why? T/I A

C. How might this investigation be related to how submarines dive underwater? T/I A

Figure 7 A Cartesian diver in action

Gay-Lussac's Law

It is the first really cold morning in early winter. You go to start your car. You notice that your tire pressure indicator is on. Yesterday the tire pressure was fine. You check your tire pressure and it is indeed below the recommended level in all four tires. What happened? As the temperature outside decreased, your tire pressure also decreased.

The data in **Table 2** were obtained by measuring the pressure of a constant volume and amount of gas as the temperature of the gas was changed. Temperature, the manipulated variable, is plotted along the x-axis (**Figure 8**). Pressure, the responding variable, is plotted along the y-axis. The amount of gas and the volume of the gas remain constant throughout this experiment. As the temperature of the gas is increased, the pressure of the gas also increases. There is a direct relationship between temperature and pressure. This relationship is now known as **Gay-Lussac's law**: the pressure of a gas increases proportionally as its temperature increases.

Table 2 Pressure versus Temperature Data for a Fixed Gas Sample

Temperature (K)	Pressure (kPa)
129	111
301	242
403	323
501	429
601	517
704	613

Gay-Lussac's law the statement that as the temperature of a gas increases, the pressure of the gas increases proportionally, provided that the volume and amount of gas remain constant; the temperature and pressure of a gas are directly proportional

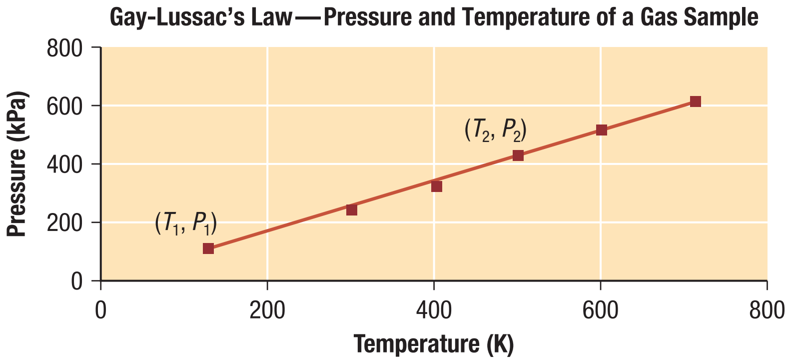

Figure 8 A graph showing the relationship between pressure and temperature for a fixed sample of gas. Volume and amount of gas are held constant.

As in the case of Boyle's law, we may analyze two data points from the plot of pressure versus temperature: (T_1, P_1) and (T_2, P_2). The values of these points are (129, 111) and (501, 425) respectively. If we divide the value of pressure, P (y value), by temperature, T (x value), for these two points, we obtain the following results:

For (T_1, P_1)

$$\frac{P_1}{T_2} = \frac{111 \text{ kPa}}{129 \text{ K}} = 0.86 \text{ kPa/K}$$

For (T_2, P_2)

$$\frac{P_2}{T_2} = \frac{429 \text{ kPa}}{501 \text{ K}} = 0.86 \text{ kPa/K}$$

Since the above ratios are equal we can conclude that

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ and } \frac{PT}{T} = k \text{ (constant)}$$

This is the mathematical expression of Gay-Lussac's law.

Gay-Lussac's Law

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}, \text{ or } PT = k \text{ (constant)}$$

The pressure of a gas is directly proportional to its temperature when the amount of gas and volume remain constant.

From our understanding of kinetic molecular theory we know that entities have greater kinetic energy at higher temperatures. With greater kinetic energy, the molecules are more likely to collide with other molecules and with the walls of the container. The result is increased pressure from more frequent collisions with the walls of the container.

Tutorial 1 Using the Gas Laws

The gas laws that we have discussed thus far—Charles' law, Boyle's law, and Gay-Lussac's law—are extremely useful for predicting volumes, temperatures, and/or pressures when various changes have been made to a sample of gas. This tutorial will help you to solve problems involving these gas laws.

Sample Problem 1: Pressure and Volume

A weather balloon is filled with 60.0 L of hydrogen gas at sea level pressure (101.3kPa). It then rises to 900 m above Earth's surface. The atmospheric pressure at this altitude is 90.6 kPa. What is the volume of the balloon at this altitude? Assume there is no change in temperature or amount of gas.

Given: Identify the known variables. Note which variables are being held constant.

initial pressure, $P_1 = 101.3$ kPa

final pressure, $P_2 = 90.6$ kPa

initial volume, $V_1 = 60.0$ L

The amount of gas and the temperature remain constant.

Required: final volume, V_2

Analysis: Use Boyle's law.

$$P_1V_1 = P_2V_2$$

Solution:

Step 1. Rearrange the equation to isolate the unknown variable.

$$V_2 = \frac{P_1V_1}{P_2}$$

Step 2. Substitute given values (including units) into the equations and solve.

$$V_2 = \frac{101.3\ \cancel{\text{kPa}} \times 60.0\ \text{L}}{90.6\ \cancel{\text{kPa}}}$$

$$= 67.1\ \text{L}$$

Statement: The volume of the balloon is 67.1 L at 900 m altitude. Check your answer. Does it make sense? Are the units appropriate?

Sample Problem 2: Pressure and Temperature

A sample of gas is stored in a reinforced steel container at −115 °C, at a pressure of 39.9 kPa. If the pressure reaches 60.8 kPa, what is the final Celsius temperature?

Given: initial pressure, $P_1 = 39.9$ kPa

final pressure, $P_2 = 60.8$ kPa

initial temperature, $t_1 = -115$ °C

The volume and amount of gas remain constant.

Required: final temperature, T_2

Analysis: Use Gay-Lussac's law.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Solution:

Step 1. Convert temperature values to kelvins.

$$T_1 = t_1 + 273$$

$$= -115 + 273$$

$$T_1 = 158\ \text{K}$$

Step 2. Rearrange the equation to isolate the unknown variable.

$$T_2 = \frac{P_2T_1}{P_1}$$

Step 3. Solve the equation (including units).

$$T_2 = \frac{60.8 \cancel{\text{kPa}} \times 158 \text{ K}}{33.9 \cancel{\text{kPa}}}$$

$$T_2 = 241 \text{ K}$$

Step 4. Convert the final temperature to the required units.

$$t_2 = T_2 - 273$$
$$= 241 - 273$$
$$t_2 = -32 \text{ °C}$$

Statement: The temperature of the gas in the container is −32 °C.

Practice

SKILLS HANDBOOK A6

1. Helium gas has a volume of 8.25 L at 446 kPa. What pressure must be applied to the gas when it occupies 12 L? T/I [ans: 307 kPa]
2. A 1.75 L sample of ammonia gas increases in volume to 6.50 L when the pressure reaches 2.84 kPa. What was the original pressure of this gas? T/I [ans: 10.5 kPa]
3. Soccer balls are typically inflated to between 60 and 110 kPa. A soccer ball is inflated indoors with a pressure of 85 kPa at 25 °C. If it is taken outside, where the temperature on the playing field is −11.4 °C, what is the pressure of the gas inside the soccer ball? T/I [ans: 75 kPa]

The Combined Gas Law

In each of the gas laws that we have studied so far, one variable was manipulated, one was allowed to respond, and all others were kept constant (controlled).

Charles' law: $\frac{V}{T}$ = constant (n and P are controlled)

Boyle's law: PV = constant (n and T are controlled)

Gay-Lussac's law: $\frac{P}{T}$ = constant (n and V are controlled)

Now we can combine Charles' law, Boyle's law, and Gay-Lussac's law into a single law. The **combined gas law** describes the relationship between volume, temperature, and pressure for any fixed amount of gas. The combined equation is

$$\frac{PV}{T} = \text{constant}$$

In other words, the product of the pressure and volume of a gas divided by its absolute temperature is a constant as long as the amount of gas is kept constant. This relationship can be expressed in a convenient form for calculations involving changes in volume, temperature, or pressure for a fixed amount of gas.

combined gas law the statement that the product of the pressure and volume of a gas sample is proportional to its absolute temperature in kelvins

The Combined Gas Law

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \text{ or } \frac{PV}{T} = \text{constant}$$

The product of the pressure and volume of a gas divided by its absolute temperature is a constant as long as the amount of gas is kept constant.

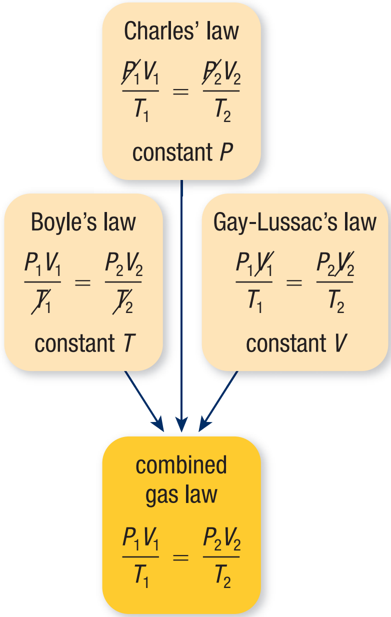

Figure 9 The combined gas law is a combination of three gas laws: Charles' law, Boyle's law, and Gay-Lussac's law. We can use it to find any of the other laws.

LEARNING TIP

Mathematical Mess

As you rearrange the combined gas law, things can get messy quickly. Be sure to take one step at a time and be careful as you perform mathematical operations. Do a quick check to make sure you rearranged the equation correctly. If you include the units in the rearranged equation, you should be left with the correct units after the others have cancelled out.

Tutorial 2 Using the Combined Gas Law

The combined gas law is very useful in predicting the effect of changes in two of the gas variables on the third when the amount of gas remains constant. In addition you can use the combined gas law to derive any of the other three gas laws that we have examined **(Figure 9)**.

Sample Problem 1: Using the Combined Gas Law

A sample of carbon dioxide gas, CO_2(g), occupies a volume of 25.0 L when the pressure is 125 kPa and the temperature is 25 °C. Calculate the volume occupied by this same quantity of carbon dioxide at STP.

Given: Identify the known variables. Decide whether the known variables are the initial or final values. Note which variables are being held constant. Remember that STP stands for standard temperature and pressure and is equal to a temperature of 25 °C and 101.3 kPa of pressure.

initial pressure, $P_1 = 125$ kPa

initial volume, $V_1 = 25.0$ L

initial temperature, $t_1 = 25$ °C

final pressure, $P_2 = 101.3$ kPa

final temperature, $T_2 = 273$ K

Amount of gas remains constant.

Required: final volume, V_2

Analysis: Use the combined gas law.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Solution:

Step 1. Convert temperature values to kelvins.

$$T_1 = 25 + 273$$
$$= 298 \text{ K}$$

Step 2. Rearrange the equation to isolate the unknown variable.

$$V_2 = \frac{P_1V_1T_2}{P_2T_1}$$

Step 3. Substitute given values (including units) into the equations and solve.

$$V_2 = \frac{125 \text{ kPa} \times 25.0 \text{ L} \times 273 \text{ K}}{101.3 \text{ kPa} \times 298 \text{ K}}$$

$$V_2 = 28.3 \text{ L}$$

Statement: The volume of the gas at STP is 28.3 L.

Practice

SKILLS HANDBOOK A6

1. A balloon at the top of Mount Logan occupies a volume (V_1) of 775 mL at a temperature of −28 °C (T_1) and a pressure of 92.5 kPa (P_1). What is the pressure (P_2) at the bottom of the mountain if the same balloon has a volume (V_2) of 825 mL at a temperature (T_2) of 15 °C? T/I [ans: 102 kPa]
2. A researcher heated a 2.75 L sample of helium gas at 99.0 kPa from 21.0 °C to 71.0 °C, and recorded that the pressure changed to 105 kPa. Calculate the final volume of the gas. T/I [ans: 3.03 L]
3. A 450 mL sample of propane gas at 253 kPa and 15 °C was compressed to 310 mL at a pressure of 405 kPa. Calculate the final temperature in Celsius. T/I [ans: 45 °C]

Research This

Investigating Gas Law Simulations

Skills: Researching, Evaluating, Communicating, Defining the Issue, Identifying Alternatives

SKILLS HANDBOOK A5.1

Animations and simulations are useful tools for exploring the gas laws. Simulations allow us to "see" gas molecules and how they behave under various situations (**Figure 10**). In this activity you will explore and evaluate several online simulations.

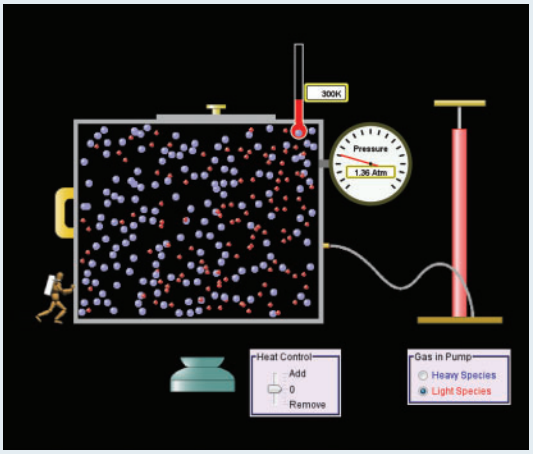

Figure 10

1. Working in a small group, develop criteria to evaluate some online simulations. Create a checklist based on these criteria.
2. Find several simulations for the same gas law.
3. Try each of these simulations, using your checklist to evaluate them.

A. What were the three most important criteria to evaluate the simulations? Briefly describe why these considerations were chosen. T/I A

B. Analyze your collected data. Which online simulation would you recommend? Explain your choice. T/I A

C. What improvements would you like made to the online simulations you evaluated? A

D. In your opinion, how does using these simulations enhance your understanding of gas laws? Explain your answer. K/U

GO TO NELSON SCIENCE

11.9 Summary

- Boyle's law states that as the volume of a sample of gas decreases, the pressure increases as long as temperature and amount of gas remain constant. Pressure and volume are inversely related.

  $P_1V_1 = P_2V_2$, or $PV = \text{constant}$

- Gay-Lussac's law states that the pressure of a gas is directly related to the temperature of the gas when the volume and amount of gas remain constant.

  $\frac{P_1}{T_1} = \frac{P_2}{T_2}$, or $\frac{P}{T} = \text{constant}$

- The combined gas law encompasses Charles' law (Section 11.8), Boyle's law, and Gay-Lussac's law for a controlled amount of gas.

  $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$, or $\frac{PV}{T} = \text{constant}$

11.9 Questions

1. Create a concept map of the four gas laws addressed in this chapter. Include descriptions of the laws, their mathematical equations, and the appropriate SI units. K/U C
2. A controlled amount of nitrogen gas initially occupies a volume of 45.2 L. Find the final volume of the nitrogen gas if it is heated from an initial temperature of 280 K to a final temperature of 560 K.
3. The air inside a tire exerts a pressure of 340 kPa at a temperature of 15° C. What will be the new pressure if the air is cooled to a temperature of −15° C? T/I
4. Using the data in **Table 3**, use the appropriate gas law to solve for the unknown quantities. T/I

Table 3 Data for a Constant Amount of Gas

	P_1	V_1	T_1	P_2	V_2	T_2
(a)	101.3 kPa	3.50 L	320 K	?	8.40 L	320 K
(b)	210 kPa	?	415 K	420 kPa	120 mL	415 K
(c)	720 mm Hg	345 mL	420 K	620 mm Hg	?	640 K

5. Neon gas is used in glass tubes to make "neon" lights. The glass must be able to withstand great variations of temperature. One night the temperature is 6.50 °C and the neon gas has a pressure of 130 kPa. The sign is turned on and the neon reaches 670.0 °C. What is the new pressure of the neon gas at this temperature? T/I
6. A compressed-gas tank contains air at 95 kPa at a temperature of 22 °C. The tank can sustain a maximum pressure of 350 kPa. What is the maximum temperature that the tank can withstand? T/I
7. Sometimes biological samples are stored at low temperatures under nitrogen gas. A biological sample in a sealed vessel contains a small volume of nitrogen gas at 101.3 kPa at 20 °C. The sample is stored at –15 °C. What is the final pressure of the nitrogen gas? T/I
8. A glass jar with a volume of 215 cm^3 is designed to withstand a pressure of 253 kPa. The jar is filled with gas at a pressure of 152 kPa at ambient temperature. (See to Section 11.7.) Can the glassware withstand the gas being heated to 200 °C? T/I
9. Your tires are adjusted to 227.5 kPa at 10 °C in the mechanic's garage. You then take your car home and park it outside. The overnight temperature drops to –5 °C. K/U T/I A
   (a) Would you expect the tire pressure to increase or decrease? Explain your answer using the kinetic molecular theory.
   (b) Determine the new tire pressure.
10. A helium balloon escapes from a birthday party. It contained 1250 cm^3 of helium at a pressure of 125 kPa. As the balloon rises it experiences a decrease in pressure to 97 kPa. Assuming no change in temperature, what would be the volume of the balloon at this altitude? T/I
11. Pressurized hydrogen gas is being tested as a fuel for vehicles. Hydrogen used to fuel these cars is stored at great pressure. In one storage tank, 4.50×10^4 L of hydrogen is stored at 2.7574×10^4 kPa. What would be the new pressure if the gas were transferred to a new tank with a volume of 6.00×10^4 L? T/I A
12. We store and use gases under pressure every day. T/I A
    (a) List three familiar examples of gases stored under pressure.
    (b) For each of your examples, list the precautions that you should take when using this product.
    (c) Explain the reason for each of these precautions, referring to the gas laws.
13. A 8.0 mL bubble of gas is released at the bottom of the ocean where the pressure is 685 kPa and the temperature is 12 °C. Calculate the volume of the gas bubble when it reaches the surface of the water, where the pressure is 99.0 kPa and the temperature is 26 °C. T/I
14. A balloon containing helium at 25 °C and a pressure of 112 kPa has a volume of 5.50 L. Calculate the volume of the balloon after it rises 8 km into the upper atmosphere, where the temperature is –34 °C and the outside air pressure is 32.0 kPa. Assume that no helium escapes and that the balloon is free to expand so that the gas pressure within it remains equal to the air pressure outside. T/I
15. A sample of carbon dioxide gas occupies a volume of 893 mL at 44 °C and 116 kPa. At what temperature will this gas occupy a volume of 1.03 L if the pressure is reduced to 102 kPa? T/I