12.2

Ideal Gases and the Ideal Gas Law

Ideally, we always arrive on time for our appointments, with everything we need in good order. Ideally, we always prepare for upcoming challenges. Ideally, we always treat one another with respect and our classroom or workplace is a harmonious, exciting, and wonderful place to be. How does the ideal compare with reality?

An Ideal Gas

Our ideas about how gases behave have been based, so far, on the gas laws. We have used the main points of the kinetic molecular theory to explain these ideas. All laws and calculations have been based on the assumption that the gases behave "ideally." Do you think this is actually the case? Just what is an "ideal gas"?

An ideal gas has the following properties:

ideal gas a hypothetical gas composed of particles that have no size, travel in straight lines, and have no attraction to each other (no intermolecular forces)

- All entities of an **ideal gas** have high translational energy, moving randomly in all directions in straight lines.
- When ideal gas entities collide with each other or with the container walls, the collisions are perfectly elastic. (There is no loss of kinetic energy.)
- The volume of an ideal gas entity is insignificant (zero) compared to the volume of the container.
- There are no attractive or repulsive forces between ideal gas entities.
- Ideal gases do not condense into liquids when cooled.

There is actually no such thing as an ideal gas. The ideal is an imaginary standard to which the behaviour of a known gas is compared. This imaginary gas has molecules with no volume and no mutual attraction to one another. It is simply a convenient approximation—or model—that works very well as we try to predict the behaviour of gases. At ordinary conditions, most gases obey the gas laws fairly well and their behaviour resembles that of an ideal gas.

We will continue—for now—with the assumption that gases behave ideally. Assuming this, we can put all our observations and gas laws together into a single equation known as the ideal gas law.

Putting It All Together: The Ideal Gas Law

In order to develop the ideal gas law, we will need to consider Charles' law, Avogadro's law, and Boyle's law as mathematical relationships:

Charles' law: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ (at constant n and P)

$$\frac{V}{T} = k \text{ or } V = kT$$

Boyle's law: $P_1V_1 = P_2V_2$ (at constant n and T)

$$PV = a \text{ or } V = \frac{a}{P}$$

Avogadro's law: $\frac{V_1}{n_1} = \frac{V_2}{n_2}$ (at constant P and T)

$$\frac{V}{n} = b \text{ or } V = bn$$

(Note that we do not need to consider Gay-Lussac's law, as it does not introduce any new variables.)

These three relationships show that the volume of a gas depends on the temperature, pressure, and amount of gas present. The letters k, a, and b are constants. We can now combine these relationships into one equation, as follows:

$V = R \times \frac{Tn}{P}$, where R is a single constant incorporating k, a, and b.

UNIT TASK **BOOKMARK**

As you work on your Unit Task on page 616, think about which gas laws researchers must consider when developing carbon capture and storage technologies.

The constant that turns all of the individual gas laws into a single equation is known as the **universal gas constant**. It is represented in the equation as R.

When pressure is measured in kilopascals (kPa), volume in litres (L), and the amount of a gas mol in moles (mol), the value of R is $8.314\ \text{kPa} \cdot \text{L} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$.

If we rearrange the above equation, we arrive at the most common form of the **ideal gas law** (**Figure 1**).

universal gas constant (R) the constant in the ideal gas law equation that relates the pressure, volume, amount, and temperature of an ideal gas

ideal gas law the statement that the product of the pressure and volume of a gas is directly proportional to the amount and the absolute temperature of the gas; $pV = nRT$

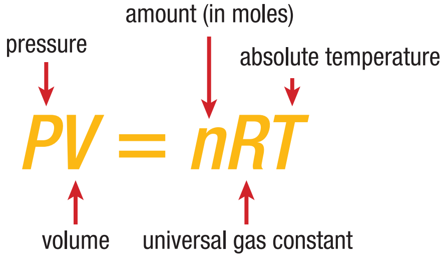

Figure 1 The ideal gas law

The ideal gas law states that the product of pressure and volume of a gas is equal to the product of the amount of gas, the universal gas constant, and the absolute temperature.

Ideal Gas Law

$$PV = nRT$$

The product of pressure and volume is equal to the product of the amount, the universal gas constant, and the absolute temperature.

Using the Ideal Gas Law

We can use the ideal gas law to find an unknown variable if the three other variables are known. While the other gas laws all compare two sets of variables, the ideal gas law works for one set of data.

LEARNING **TIP**

R **and the Ideal Gas Law**

Look closely at the units of R and you will see that they represent the variables in the ideal gas law: "kPa" for pressure, "L" for volume, and so on.

Tutorial **1** Using the Ideal Gas Law

As you work through this tutorial, you will see that we can extend the use of the ideal gas law equation to include and/or determine other variables. For example, we can adapt the ideal gas law to consider the molar mass of a gas or its density. You already know that the amount of a substance (n) is calculated from its mass (m) and its molar mass (M):

$$n = \frac{m}{M}$$

You also know that density (d) is calculated by dividing its mass (M) by its volume (V):

$$d = \frac{m}{V}$$

When solving problems using the ideal gas law, make sure that your variables are in the correct units. For example, if $R = 8.314\ \text{kPa} \cdot \text{L} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$, then P must be in kPa, V must be in L, n must be in mol, and T must be in K. When you have solved the problem, check your answer. Does it make sense? Are the units appropriate?

Sample Problem 1: Determining Amount Using the Ideal Gas Law

The Goodyear blimp has a volume of 2.5×10^7 L and usually operates with the gas at a temperature of 12 °C and a pressure of 112 kPa (**Figure 2**). What amount of helium does the blimp contain? What is the mass of this amount of helium?

Figure 2 Blimps generate lift by using gases that are lighter than air. Although hydrogen is lighter than helium, it is no longer used because it is flammable.

Given: $V = 2.5 \times 10^7$ L

$t = 12$ °C

$P = 112$ kPa

$R = 8.314$ kPa • L • mol^{-1} • K^{-1}

$M_{He} = 4.0$ g/mol

Required: amount of helium, n_{He}; mass of helium, m_{He}

Analysis: Use the ideal gas law equation to calculate the amount of gas in the blimp.

$$PV = nRT$$

Following this, convert the amount to mass.

Solution:

Step 1. Convert the temperature value(s) to kelvins.

$$T = 12 + 273$$
$$= 285 \text{ K}$$

Step 2. Rearrange the ideal gas law equation to find the amount of gas.

$$n = \frac{PV}{RT}$$

Step 3. Substitute known values into the equation and solve.

$$n = \frac{112 \cancel{\text{kPa}} \times 2.5 \times 10^7 \cancel{\text{L}}}{8.314 \cancel{\text{kPa}} \cdot \cancel{\text{L}} \cdot \text{mol}^{-1} \cdot \cancel{\text{K}^{-1}} \times 285 \cancel{\text{K}}}$$
$$= 1.18 \times 10^6 \text{ mol}$$

Step 4. Convert amount into mass using the appropriate conversion factor involving molar mass.

$$m_{He} = n_{He} \times \frac{4.0 \text{ g}}{1 \text{ mol}}$$
$$= 1.18 \times 10^6 \cancel{\text{mol}} \times \frac{4.0 \text{ g}}{1 \cancel{\text{mol}}}$$
$$m_{He} = 4.7 \times 10^6 \text{ g}$$

Statement: The blimp contains 1.2×10^6 mol of helium, with a mass of 4.7×10^6 g.

Sample Problem 2: Determining Volume Using the Ideal Gas Law

Calculate the volume of 32.4 g of nitrogen gas, $N_2(g)$, in a container at 25 °C and 96.4 kPa.

Given: $t = 25$ °C

$P = 96.4$ kPa

$R = 8.314$ kPa • L • mol^{-1} • K^{-1}

$m_{N_2} = 32.4$ g

$M_{N_2} = 28.02$ g/mol

Required: volume of nitrogen gas, V

Analysis: Use the ideal gas law.

$$PV = nRT$$

Solution:

Step 1. Convert the temperature value(s) to kelvins.

$$T = 25 + 273$$
$$= 298 \text{ K}$$

Step 2. Convert mass into amount using the appropriate conversion factor involving molar mass. Remember that we are considering nitrogen gas, which consists of diatomic molecules.

$$n_{N_2} = m_{N_2} \times \frac{1 \text{ mol}}{28.02 \text{ g}}$$

$$= 32.4 \cancel{\text{g}} \times \frac{1 \text{ mol}}{28.02 \cancel{\text{g}}}$$

$$n_{N_2} = 1.156 \text{ mol}$$

Step 3. Rearrange the ideal gas law equation to isolate the required variable.

$$V = \frac{nRT}{P}$$

Step 4. Solve the equation (including units).

$$V = \frac{1.156 \cancel{\text{mol}} \times 8.314 \cancel{\text{kPa}} \cdot \text{L} \cdot \cancel{\text{mol}^{-1}} \cancel{\text{K}^{-1}} \times 298 \cancel{\text{K}}}{96.4 \cancel{\text{kPa}}}$$

$$= 29.7 \text{ L}$$

Statement: The container has a volume of 29.7 L.

Sample Problem 3: Determining Density Using the Ideal Gas Law

Determine the density of 1.00 mol of pure carbon dioxide gas at STP.

Given: $T = 273 \text{ K}$

$P = 101.3 \text{ kPa}$

$R = 8.314 \text{ kPa} \cdot \text{L} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$

$M_{CO_2} = 44.01 \text{ g/mol}$

$n_{CO_2} = 1.00 \text{ mol}$

Required: density of carbon dioxide gas, d_{CO_2}

To find the density, you first have to determine the mass, m, and the volume, V.

Analysis: First use the ideal gas law to determine the volume.

$$PV = nRT$$

Next, convert the given amount of gas to mass.

Finally, use the expression $d = m/V$ to calculate the density of the gas.

Solution:

Step 1. Rearrange the ideal gas law equation to isolate the unknown variable, V.

$$V = \frac{nRT}{P}$$

Step 2. Substitute known values into the equation and solve.

$$V = \frac{1.00 \cancel{\text{mol}} \times 8.314 \cancel{\text{kPa}} \cdot \text{L} \cdot \cancel{\text{mol}^{-1}} \cdot \cancel{\text{K}^{-1}} \times 273 \cancel{\text{K}}}{101.3 \cancel{\text{kPa}}}$$

$$= 22.4 \text{ L}$$

You may recall seeing this number before!

Step 3. Convert amount of CO_2 into mass of CO_2 using the appropriate conversion factor involving molar mass.

$$n_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}}$$

$$m_{CO_2} = n_{CO_2} \times \frac{44.01 \text{ g}}{\text{mol}}$$

$$= 1.00 \cancel{\text{mol}} \times \frac{44.01 \text{ g}}{1 \cancel{\text{mol}}}$$

$$m_{CO_2} = 44.01 \text{ g}$$

Step 4. Use the relationship $d_{CO_2} = m/V$ to determine the density of CO_2.

$$d_{CO_2} = \frac{m}{V}$$

$$= \frac{44.01 \text{ g}}{22.4 \text{ L}}$$

$$d_{CO_2} = 1.96 \text{ g/L}$$

Statement: The density of pure carbon dioxide gas at STP is 1.96 g/L.

Sample Problem 4: Determining Molar Mass Using the Ideal Gas Law

10.24 g of a pure gas occupies 2.10 L at 123 °C and 99.7 kPa. Calculate the molar mass of this gas.

Given: $t = 123$ °C

$V = 2.10$ L

$P = 99.7$ kPa

$R = 8.314 \text{ kPa} \cdot \text{L} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$

$m = 10.24$ g

Required: amount of gas, n; molar mass of gas, M

Analysis: First, use the ideal gas law to find the amount of gas.

$$PV = nRT$$

Next, use the amount of gas to determine the molar mass.

Solution:

Step 1. Convert the temperature value(s) to kelvins.

$$T = 123 + 273$$

$$= 396 \text{ K}$$

Step 2. Rearrange the ideal gas law equation to isolate the unknown variable, n.

$$n = \frac{PV}{RT}$$

Step 3. Substitute known values into the equation and solve.

$$n = \frac{99.7 \cancel{\text{kPa}} \times 2.10 \cancel{\text{L}}}{8.314 \cancel{\text{kPa}} \cdot \cancel{\text{L}} \cdot \text{mol}^{-1} \cdot \cancel{\text{K}^{-1}} \times 396 \cancel{\text{K}}}$$

$$= 0.06359 \text{ mol}$$

Step 4. Find the molar mass of the gas by rearranging the values for mass and amount to give a value with the units g/mol.

$$M = \frac{10.24 \text{ g}}{0.06359 \text{ mol}}$$

$$= 161 \text{ g/mol}$$

Statement: The molar mass of the gas is 161 g/mol.

Practice

SKILLS HANDBOOK A6

1. A neon gas in a sign has a volume of 42.5 L at 25 °C and 3.5 kPa. Calculate the mass of neon. T/I [ans: 1.2 g]
2. What amount of ethane gas is present in a sample that has a volume of 320 mL at 25.0 °C and 420 kPa? T/I [ans: 0.054 mol]
3. A tank contains 40 kg of propane gas, $C_3H_8(g)$, at a pressure of 170 kPa and a temperature of 25 °C. What is the volume of the tank? T/I [ans: 1.3×10^4 L]
4. A certain diatomic gas is found to have a density of 3.17 g/L at STP. Calculate the molar mass of this gas. T/I [ans: 71.0 g/mol]

Molar Volume for Ideal Gases and Real Gases

We will now revisit the concept of molar volume. We can use the ideal gas law to determine the molar volume of a gas at any temperature and pressure. To find the volume, V, at STP, we simply rearrange the equation and substitute in the known variables. Remember that the molar volume is the volume occupied by one mole of an ideal gas at STP (or SATP).

$$V = ?$$

$$t = 0\ °\text{C}$$

$$\begin{aligned} T &= 0 + 273\ \text{K} \\ &= 273\ \text{K} \end{aligned}$$

$$P = 101.3\ \text{kPa}$$

$$n = 1.00\ \text{mol}$$

$$R = 8.314\ \text{kPa} \cdot \text{L} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$$

$$PV = nRT$$

$$\begin{aligned} V &= \frac{nRT}{P} \\ &= \frac{1.00\ \cancel{\text{mol}} \times 8.314\ \cancel{\text{kPa}} \cdot \text{L} \cdot \cancel{\text{mol}^{-1}} \cdot \cancel{\text{K}^{-1}} \times 273\ \cancel{\text{K}}}{101.3\ \cancel{\text{kPa}}} \\ &= 22.4\ \text{L} \end{aligned}$$

Thus, the molar volume of an ideal gas at STP is 22.4 L. This value appeared in Sample Problem 3 in the previous tutorial. This is also the value that was described in Section 12.1. This calculation shows how the molar volume can be determined mathematically.

Deviations from the Ideal Gas Law—Real Gases

If we analyze experimental molar volumes for different gases at STP, we see that they vary slightly from the ideal value of 22.4 L (**Table 1**). Such data remind us that, when we use the ideal gas equation, we are *imagining* ideal gases. When we experimentally measure the actual molar volume of a gas at STP, we are working with real gases.

Deviations from the gas laws occur because ideal gases do not really exist. Under ordinary circumstances, the differences between ideal gas behaviour and real gas behaviour are extremely small. Under high pressures and low temperature conditions, however, the behaviour of gases deviates more significantly.

As you know, the compressed butane in a lighter is in a liquid state. So is compressed propane in a barbecue tank. Clearly, real gases do condense into liquids (or solids) and gas molecules do experience intermolecular forces, and thus they do attract each other.

At normal atmospheric pressures, there are relatively few collisions between the entities and therefore the gases behave fairly ideally. At high pressures, however, real

Table 1 Actual Molar Volumes of Various Real Gases at STP

Gas	Molar Volume at STP (L/mol)
any ideal gas	22.414
helium, He	22.426
oxygen, O_2	22.397
chlorine, Cl_2	22.063
ammonia, NH_3	22.097

gas entities experience more collisions. This increases the attraction between the entities. As well, when the volume of the gas is decreased to much smaller volumes, the volume of the gas entities is significant with respect to the size of the container.

At normal temperatures, gas entities have a lot of kinetic energy and therefore travel at relatively high speeds. These high speeds make the forces of attraction insignificant when the entities collide. At low temperatures, however, real gas entities travel relatively slowly. At these reduced speeds, real gas entities experience forces of attraction when they collide.

The behaviour of gases resembles ideal gas behaviour when pressures are fairly low, volumes are relatively large, and temperatures are relatively high. Under these conditions the rapid movement of the gas entities overcomes any attractive forces between them, and the volume of the entities is small compared with the size of the container.

Despite these differences, we can use the ideal gas law to predict the behaviour and properties of gases under most circumstances. It is used extensively in laboratories and industrial applications. In fact, many scientific theories consider ideal situations that have to be modified to be applied to the real world.

CAREER LINK

To find out more about how engineers, such as pneumatics engineers, work with real gas, and adapt the ideal gas law to real situations,

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12.2 Summary

- Ideal gas molecules behave ideally. They do not condense to a liquid when cooled, they have no volume, and they do not attract each other.
- The ideal gas law, $PV = nRT$, is a combination of Charles' law, Boyle's law, and Avogadro's law (**Figure 3**). It is a powerful relationship that describes the behaviour of ideal gases.
- The gas constant (R) can be determined experimentally and has a value of $8.314\ \text{kPa} \cdot \text{L} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$.
- Even though there are no ideal gases in the real world, we can use the ideal gas laws to predict the behaviour of gases under most situations.

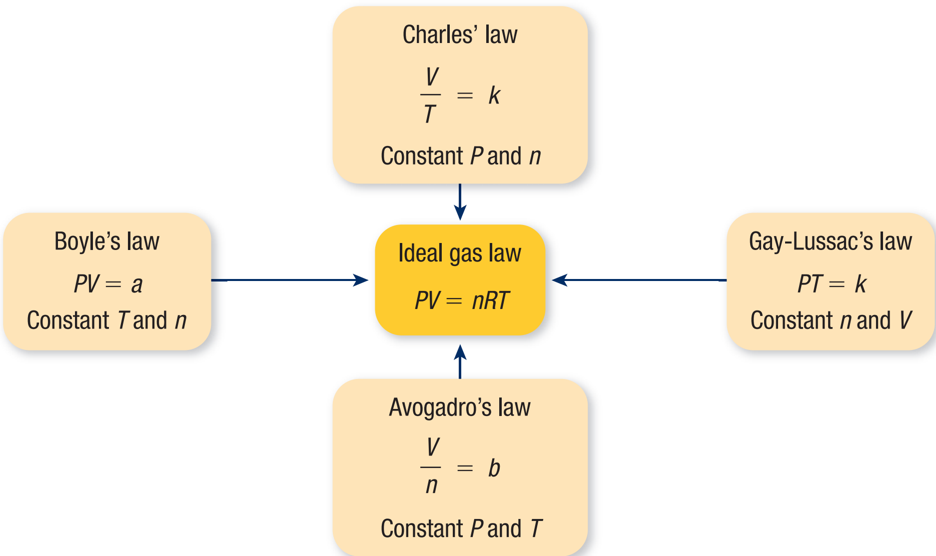

Figure 3 We can combine the individual gas laws to form the ideal gas law.

12.2 Questions

1. Describe what is meant by the term "ideal gas." K/U
2. Under what conditions is a gas closest to having the properties of an ideal gas? Explain why. K/U
3. (a) Distinguish between the molar mass of a gas and the molar volume of a gas. K/U
   (b) Does molar mass and/or molar volume change with temperature and pressure?
4. Use the ideal gas law to find the volume that 4.30 mol of oxygen gas occupies at 99.7 kPa and 35.0 °C. T/I
5. Laughing gas (dinitrogen monoxide), $N_2O(g)$, is sometimes used by dentists to keep patients relaxed during dental procedures. A 47.5 L cylinder of laughing gas has a pressure of 112.0 kPa and the temperature of the dental office is 22.0 °C. Determine the mass of laughing gas in the cylinder. T/I
6. A sample of helium gas occupies 4.52 L at a pressure of 45 kPa. If the gas has a mass of 43.5 g, calculate the temperature of the gas in degrees Celsius. T/I
7. A sample of 7.76 g of nitrogen gas, $N_2(g)$, is stored in a 3.50 L container at a temperature of 35 °C. Under what pressure is the gas being stored? T/I
8. A sample of gas has a mass of 7.60 g and occupies a volume of 2.45 L at STP. T/I
   (a) Calculate the molar mass of the gas.
   (b) Is it possible to identify the gas from these data? Explain.
9. (a) Calculate the molar volume of any gas at 112 kPa and 22 °C.
   (b) What assumptions did you make in your calculation? K/U T/I
10. Find the density of propane gas, C_3H_8,(g), at SATP. If the density of air (a mixture of gases) at SATP is approximately 1.2 g/L, will pure gaseous C_3H_8 sink or rise in air? Should propane gas detectors be placed up high or down low in a room? T/I A
11. Give three examples of how the gas laws apply to the operation and safety of automobiles. A
12. You are trying to experimentally determine the molar mass of a gas in order to identify it. What values must you be able to calculate to determine the molar mass of this mystery gas? T/I
13. (a) Under what circumstances do real gases deviate from ideal behaviour?
    (b) Research two applications where you think the differences between real gas and ideal gas behaviour would be significant. K/U A

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