9.5 Stoichiometry of Solutions

Figure 1 Chemists frequently work with aqueous solutions because they can control many variables by adjusting the temperature and/or concentrations.

Picture a stereotypical chemist at work. There are probably several bottles and jars of mysterious liquids in your mental image. It might be a stereotype, but a great deal of chemistry really does take place in aqueous solutions (**Figure 1**). This is partly because water is cheap and easily accessible. It is also good at dissolving many substances that it comes in contact with. Chemicals mix more completely when they are dissolved, resulting in faster reactions. Allowing reactions to occur in solution also makes it easier to control how quickly the reactions occur. The investigator can adjust the solution temperature or the reactant concentrations.

Being able to predict the quantity of product or the quantity of reactants required is critical to the success of a chemical process. For example, in Section 9.1, you learned that patients consume a "barium meal" prior to having X-rays taken of their gastrointestinal tract. The barium sulfate used can be prepared by combining solutions of barium chloride and sodium sulfate:

$$BaCl_2(aq) + Na_2SO_4(aq) \rightarrow BaSO_4(s) + 2\ NaCl(aq)$$

Barium ions, however, are quite toxic. No free barium ions should remain in the final solution. Therefore, when barium sulfate is prepared, the quantities of both reactants have to be carefully measured. Chemists must determine the correct amount of sodium sulfate required using stoichiometry.

Stoichiometry and Reactions

In Chapter 7 you learned how we can use stoichiometry to predict the quantity of one chemical required to react with another. Recall that we can use a balanced chemical equation to determine the ratios of chemicals involved in the reaction. It is likely that we will know the amount concentrations and volumes of one or more of the reactant solutions. We will make a stoichiometric prediction regarding the reaction. First, though, we must convert the known quantities of chemicals used to amounts (**Figure 2**). We can apply this concept to making a barium meal that is safe to consume.

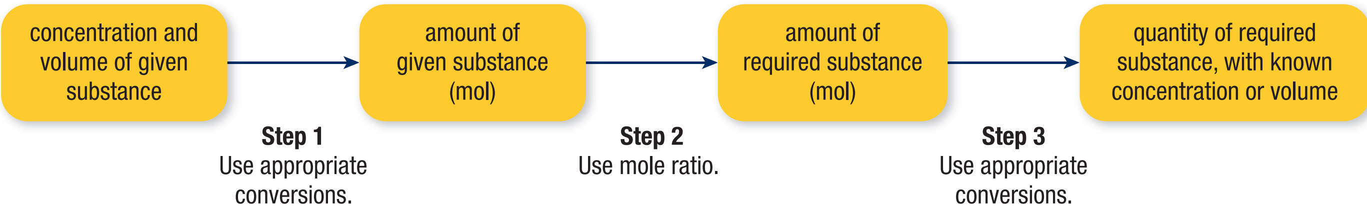

Figure 2 The general strategy for solving stoichiometry problems

Tutorial 1 Solving Stoichiometry Problems Involving Solutions

The strategies you learned to solve stoichiometric problems in Chapter 7 apply to solutions as well. The only difference is that amount concentrations and volumes of solutions are involved.

Sample Problem 1: Determining Volume to Precipitate a Compound

Determine the minimum volume of 0.42 mol/L sodium sulfate, $Na_2SO_4(aq)$, that is required to react completely with all the barium ions in 500.0 mL of a 0.100 mol/L barium chloride, $BaCl_2$, solution.

Given: $c_{Na_2SO_4} = 0.42$ mol/L; $c_{BaCl_2} = 0.100$ mol/L; $V_{BaCl_2} = 500.0$ mL

Required: volume of 0.42 mol/L sodium sulfate, $V_{Na_2SO_4}$

Analysis:

$$c = \frac{n}{V}$$

Solution:

Step 1. Convert all volumes of the solutions to litres, if necessary.

$$V_{BaCl_2} = 500.0\ \cancel{mL} \times \frac{1\ L}{1000\ \cancel{mL}}$$
$$V_{BaCl_2} = 0.5000\ L$$

Step 2. Write a balanced equation for the reaction, listing the given value(s), required value(s), and amount concentrations below the substances being considered in the problem.

$BaCl_2(aq)$	$+\ Na_2SO_4(aq)$	$\rightarrow BaSO_4(s) + 2\ NaCl(aq)$
$V_{BaCl_2} = 0.5000\ L$	$V_{Na_2SO_4}$	
$c_{BaCl_2} = 0.100\ mol/L$	$c_{Na_2SO_4} = 0.42\ mol/L$	

Step 3. Rearrange the concentration equation to solve for the amount of barium chloride, n_{BaCl_2}.

$$n_{BaCl_2} = c_{BaCl_2}V_{BaCl_2}$$
$$= 0.100\ mol/L \times 0.5000\ L$$
$$n_{BaCl_2} = 0.0500\ mol$$

Step 4. Determine the amount of the substance whose volume is required from the amount of the substance whose volume and concentration are given. To do this, multiply the amount calculated in Step 3 by a suitable conversion factor derived from the mole ratio in the balanced equation. In this case, the conversion factor is either

$$\frac{1\ mol_{BaCl_2}}{1\ mol_{Na_2SO_4}} \text{ or } \frac{1\ mol_{NaSO_4}}{1\ mol_{BaCl_2}}$$

Since we are determining the amount of Na_2SO_4 from the amount of $BaCl_2$, we use the conversion factor $\frac{1\ mol_{Na_2SO_4}}{1\ mol_{BaCl_2}}$, as follows:

$$n_{Na_2SO_4} = 0.0500\ \cancel{mol_{BaCl_2}} \times \frac{1\ mol_{Na_2SO_4}}{1\ \cancel{mol_{BaCl_2}}}$$
$$n_{Na_2SO_4} = 0.0500\ mol_{Na_2SO_4}$$

Step 5. Determine the volume of the substance whose volume is required by rearranging the amount concentration equation and substituting the values determined in Steps 2 and 4.

$$c_{Na_2SO_4} = \frac{n_{Na_2SO_4}}{V_{Na_2SO_4}}$$
$$V_{Na_2SO_4} = \frac{n_{Na_2SO_4}}{c_{Na_2SO_4}}$$
$$= \frac{0.0500\ mol}{\frac{0.42\ mol}{1\ L}}$$
$$= 0.0500\ \cancel{mol} \times \frac{1\ L}{0.42\ \cancel{mol}}$$
$$V_{Na_2SO_4} = 0.12\ L \text{ or } 120\ mL$$

Statement: The minimum volume of 0.42 mol/L sodium sulfate required to react with 500.0 mL of 0.100 mol/L of a barium chloride solution is 120 mL.

UNIT TASK **BOOKMARK**

In your analysis of your Unit Task data (p. 498), you will calculate the concentrations of cations in two different solutions. You will then use your findings to compare the solutions.

LEARNING **TIP**

Determining Amount

There is an alternative to using the conversion factor method for finding the amount of barium chloride in Step 3. You could rearrange the concentration equation and solve for *n*.

LIMITING REAGENT PROBLEMS

If the volume and concentration of both of the reactants are given, you will have to determine which of them is the limiting reagent. (Recall that the limiting reagent is the reactant that is completely used up during the reaction.) Sample Problem 2 includes this additional step.

Sample Problem 2: Predicting the Mass of Precitate Expected

Predict the mass of precipitate expected when 1.50 L of 0.800 mol/L sodium carbonate, Na_2CO_3, is mixed with 850 mL of a 1.00 mol/L aluminum nitrate, $Al(NO_3)_3$, solution.

Given: $V_{Na_2CO_3} = 1.50$ L; $c_{Na_2CO_3}$0.800 mol/L; $V_{Al(NO_3)_3} = 850$ mL; $c_{Al(NO_3)_3} = 1.00$ mol/L

Required: mass of aluminum carbonate precipitate, $m_{Al_2(CO_3)_3}$

Solution:

Step 1. Convert all volumes to litres, if necessary.

$$V_{Al(NO_3)_3} = 850\ \cancel{mL} \times \frac{1\ L}{1000\ \cancel{mL}}$$

$$V_{Al(NO_3)_3} = 0.850\ L$$

Step 2. Write a balanced equation for the reaction, listing the given value(s), required value(s), and amount concentrations below the substances being considered in the problem.

3 $Na_2CO_3(aq)$	+ 2 $Al(NO_3)_3(aq) \rightarrow$	$Al_2(CO_3)_3(s)$ + 6 $NaNO_3(aq)$
$V_{Na_2CO_3} = 1.50$ L	$V_{Al(NO_3)_3} = 0.850$ L	$m_{Al_2(CO_3)_3}$
$c_{Na_2CO_3} = 0.800$ mol/L	$c_{Al(NO_3)_3} = 1.00$ mol/L	

Step 3. Determine the amounts of both substances given using the given volumes and concentrations as follows:

$$n_{Na_2CO_3} = 1.50\ \cancel{L} \times \frac{0.800\ mol}{1\cancel{L}}$$

$$n_{Na_2CO_3} = 1.20\ mol$$

$$n_{Al(NO_3)_3} = 0.850\ \cancel{L} \times \frac{1.00\ mol}{1\ \cancel{L}}$$

$$n_{Al(NO_3)_3} = 0.850\ mol$$

Step 4. Determine which reactant is the limiting reagent. Since the amounts of two reactants are now known, one of the reactants is a limiting reagent. To determine which one is the limiting reagent, determine the amount of one reactant required to react completely with the other reactant as follows:

$$n_{Na_2CO_3} = 0.850\ \cancel{mol_{Al(NO_3)_3}} \times \frac{3\ mol_{Na_2CO_3}}{2\ \cancel{mol_{Al(NO_3)_3}}}$$

$$n_{Na_2CO_3} = 1.28\ mol$$

Therefore, 0.850 mol $Al(NO_3)_3$ requires 1.28 mol of Na_2CO_3. Since only 1.2 mol of Na_2CO_3 is present, sodium carbonate is the limiting reagent.

Step 5. Use the amount of the limiting reagent determined in Step 4 to determine the amount of the substance whose mass is required, $n_{Al_2(CO_3)_3}$.

$$n_{Al_2(CO_3)_3} = 1.2\ \cancel{mol_{Na_2CO_3}} \times \frac{1\ mol_{Al_2(CO_3)_3}}{3\ \cancel{mol_{Na_2CO_3}}}$$

$$n_{Al_2(CO_3)_3} = 0.40\ mol_{Al_2(CO_3)_3}$$

Step 6. Use the amount calculated in Step 5 to determine the mass of the substance whose mass is required, $m_{Al_2(CO_3)_3}$.

$$m_{Al_2(CO_3)_3} = 0.400\ \cancel{mol} \times \frac{233.99\ g}{1\ \cancel{mol}}$$

$$m_{Al_2(CO_3)_3} = 93.6\ g$$

Statement: Combining 1.50 L of 0.80 mol/L sodium carbonate with 0.850 L of a 1.00 mol/L aluminum nitrate produces 93.6 g of an aluminum carbonate precipitate.

Practice

SKILLS HANDBOOK A6

1. Sodium carbonate, Na_2CO_3, can be used to precipitate strontium ions from a solution of strontium chloride, $SrCl_2$:

   $SrCl_2(aq) + Na_2CO_3(aq) \rightarrow SrCO_3(s) + 2\ NaCl(aq)$

   You have 150 mL of a 0.25 mol/L strontium chloride solution. T/I

   (a) What volume of 0.500 mol/L sodium carbonate is required to precipitate all the strontium ions from this solution? [ans: 0.075 L or 75 mL]

   (b) What mass of precipitate is expected? [ans: 5.5 g]

2. A jelly-like precipitate of iron(III) hydroxide, $Fe(OH)_3$, forms when solutions of iron(III) nitrate, $Fe(NO_3)_3$, and potassium hydroxide, KOH, are combined:

   $Fe(NO_3)_3(aq) + 3\ KOH(aq) \rightarrow Fe(OH)_3(s) + 3\ KCl(aq)$

   What mass of precipitate is expected to form when 70.0 mL of 0.80 mol/L potassium hydroxide is added to 40.0 mL of a 0.50 mol/L iron(III) nitrate solution? T/I [ans: 2.0 g]

Stoichiometry of Ions in Solution

The bright glow of a conductivity tester indicates that a solution of calcium chloride is a good conductor of electricity (**Figure 3**). Our explanation is that calcium chloride completely dissociates into its ions as it dissolves (**Figure 4**):

$$CaCl_2(s) \rightarrow Ca^{2+}(aq) + 2\ Cl^-(aq)$$

1 mol   1 mol   2 mol

This equation is called a dissociation equation. It indicates that for each mole of calcium chloride in the solution, 1 mol of calcium ions and 2 mol of chloride ions dissociate from each other. For example, in a 0.25 mol/L calcium chloride solution, the concentration of calcium ions is 0.25 mol/L and the concentration of chloride ions is 0.50 mol/L.

LEARNING TIP

Ion Concentrations

The concentration of the ions is not always the same as the concentration of the compound they came from. In this case, when $CaCl_2$ dissociates, the chloride concentration will always be double the calcium chloride concentration since 9 chloride ions are released per formula unit of $CaCl_2$.

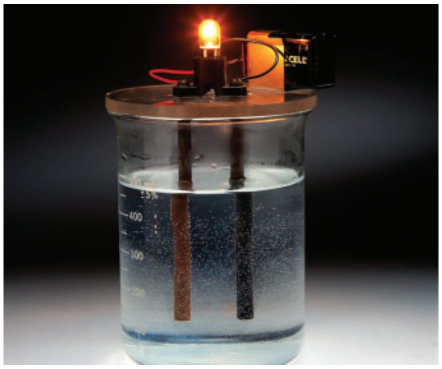

Figure 3 Calcium chloride is a good electrolyte because it dissolves in water to produce a solution that conducts electricity well.

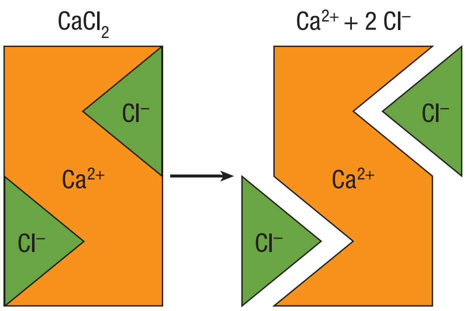

Figure 4 How do the final concentrations of chloride and calcium ions compare to the initial concentration of the compound?

Tutorial 2 Determining the Amount Concentration of Ions in Solution

When calculating the amount concentration of each type of ion released when an ionic compound dissolves, you will need to write a dissociation equation.

Investigation 9.5.1

Percentage Yield of a Precipitation Reaction (p. 453)

In this investigation you will first prepare two standard solutions, one of which contains calcium ions. You will then use the other solution to remove the calcium ions in a precipitation reaction. What is the percentage yield of the reaction?

Sample Problem 1: Calculating the Amount Concentration of Ions

A technician dissolves 17.1 g of aluminum sulfate, $Al_2(SO_4)_3$, to prepare 250.0 mL of solution. What are the amount concentrations of aluminum ions and sulfate ions in the solution?

Given: $m_{Al_2(SO_4)_3} = 17.1\text{ g}$; $V_{Al_2(SO_4)_3} = 250.0\text{ mL}$

Required: amount concentration of aluminum and sulfate ions, $c_{Al^{3+}}$; $c_{SO_4^{\ 2-}}$

Solution:

Step 1. Convert given volumes to litres, if necessary.

$$V_{Al_2(SO_4)_3} = 250.0\ \cancel{\text{mL}} \times \frac{1\text{ L}}{1000\ \cancel{\text{mL}}}$$

$$V_{Al_2(SO_4)_3} = 0.2500\text{ L}$$

Step 2. Determine the amount of solute from the given mass of solute, if necessary.

$$n_{Al_2(SO_4)_3} = 17.1\ \cancel{\text{g}} \times \frac{1\text{ mol}}{342.14\ \cancel{\text{g}}}$$

$$n_{Al_2(SO_4)_3} = 0.0500\text{ mol}$$

Step 3. Determine the amount concentration of the solution.

$$c_{Al_2(SO_4)_3} = \frac{n_{Al_2(SO_4)_3}}{V_{Al_2(SO_4)_3}} = \frac{0.0500\text{ mol}}{0.250\text{ L}}$$

$$c_{Al_2(SO_4)_3} = 0.200\text{ mol/L}$$

Step 4. Write a dissociation equation listing the calculated amounts and the required value(s) below the substances being considered in the problem.

$Al_2(SO_4)_3(s)$	$\rightarrow$	$2\ Al^{3+}(aq)$	$+\ 3\ SO_4^{\ 2-}(aq)$
$c_{Al_2(SO_4)_3} = 0.200$ mol/L		$c_{Al^{3+}}$	$c_{SO_4^{\ 2-}}$

Step 5. Convert the concentration of the compound into concentration of the ions.

$$c_{Al^{3+}} = 0.200\ \frac{\cancel{\text{mol}_{Al_2(SO_4)_3}}}{\text{L}} \times \frac{2\text{ mol}_{Al^{3+}}}{1\ \cancel{\text{mol}_{Al_2(SO_4)_3}}}$$

$$c_{Al^{3+}} = 0.400\text{ mol/L}$$

$$c_{SO_4^{\ 2-}} = 0.200\ \frac{\cancel{\text{mol}_{Al_2(SO_4)_3}}}{\text{L}} \times \frac{3\text{ mol}_{SO_4^{\ 2-}}}{1\ \cancel{\text{mol}_{Al_2(SO_4)_3}}}$$

$$c_{SO_4^{\ 2-}} = 0.600\text{ mol/L}$$

Statement: The aluminum and sulfate ion concentrations in 250.0 mL of solution containing 17.1 g of aluminum sulfate are 0.400 mol/L and 0.600 mol/L respectively.

Practice

SKILLS HANDBOOK A6

1. Find the amount concentration of the anion in each of the following solutions: T/I
   (a) 0.50 mol/L barium chloride, $BaCl_2$ [ans: 1.00 mol/L]
   (b) 6.0 mol/L potassium hydroxide, KOH [ans: 6.0 mol/L]
   (c) 0.10 mol/L aluminum chlorate, $Al(ClO_3)_3$ [ans: 0.30 mol/L]
2. Calculate the amount concentration of the ammonium ion in 100.0 mL of a solution containing 14.4 g of ammonium carbonate. T/I [ans: 3.00 mol/L]
3. Calculate the mass of sodium phosphate required to prepare 1.75 L of solution in which the sodium ion concentration is 0.25 mol/L. T/I [ans: 24 g]

9.5 Summary

- Many applications involve determining the required quantities of reactants.
- In solution stoichiometry problems, concentrations and volumes are used to determine the amount of a given chemical using the equation $n = cv$.
- Stoichiometry problems may involve determining the limiting reagent.
- In limiting reagent problems, the limiting reagent determines the amount of product expected.
- For highly soluble ionic compounds, the amount concentration of each ion released can be calculated from a dissociation equation. Ion concentration equals the amount concentration of the compound multiplied by the coefficient of that ion in the dissociation equation.

9.5 Questions

1. Combining solutions of silver nitrate and sodium chloride produces a silver chloride precipitate:

   $AgNO_3(aq) + NaCl(aq) \rightarrow AgCl(s) + NaNO_3(aq)$

   A researcher discovered that 32.0 mL of 0.100 mol/L silver nitrate is required to precipitate all the chloride ions in 25 mL of a solution of sodium chloride. T/I
   (a) What is the amount concentration of sodium ions in the initial sodium chloride solution?
   (b) What is the concentration, in g/L, of sodium chloride in the initial sodium chloride solution?
2. Nickel ions can be removed from a nickel(II) sulfate solution, $NiSO_4(aq)$, by precipitating them with a solution of sodium hydroxide, NaOH(aq). T/I C
   (a) Write the balanced chemical equation for this reaction.
   (b) Predict the mass of precipitate expected when 50.0 mL of 0.45 mol/L nickel(II) sulfate solution is combined with 25.0 mL of 1.00 mol/L sodium hydroxide solution.
3. Combining solutions of sodium carbonate and calcium chloride produces a calcium carbonate precipitate:

   $Na_2CO_3(aq) + CaCl_2(aq) \rightarrow CaCO_3(s) + 2\ NaCl(aq)$

   (a) During an investigation, 15.2 g of calcium carbonate was collected using 200.0 mL of sodium carbonate solution and an excess of calcium chloride solution. What was the amount concentration of the original sodium carbonate solution?
   (b) What volume of a 0.500 mol/L calcium chloride solution would produce the same mass of precipitate as in (a)? T/I
4. The steel industry uses large volumes of concentrated hydrochloric acid to remove rust, which is essentially Fe_2O_3, from the surface of steel. This process is called "pickling:"

   $Fe_2O_3(s) + 6\ HCl(aq) \rightarrow 2\ FeCl_3(aq) + 3\ H_2O(l)$

   What volume of 12.0 mol/L HCl(aq) is required to remove 224 g of iron(III) oxide? T/I
5. When aluminum metal is placed in copper(II) sulfate solution, the aluminum ions displace the copper(II) ions in a single displacement reaction. T/I C
   (a) Write the chemical equation for the reaction.
   (b) What mass of aluminum is required to remove all the copper ions from 150 mL of a 0.100 mol/L solution of copper(II) sulfate?
6. Sodium hydroxide, NaOH(aq), is used in the production of paper, textiles, cleaners, and detergents. Sodium hydroxide is produced industrially by passing electricity through a concentrated sodium chloride solution. The chemical equation for this reaction, which involves water, is

   $2\ NaCl(aq) + 2\ H_2O(l) \rightarrow 2\ NaOH(aq) + Cl_2(g) + H_2(g)$

   As much as 45×10^6 t of sodium hydroxide is produced around the world each year. What volume of a 6.0 mol/L sodium chloride solution is required to produce this mass of sodium hydroxide? T/I
7. An unknown solution was prepared by dissolving 0.42 g of sodium chloride, NaCl(s), or calcium chloride, $CaCl_2(s)$, in enough water to make 100.0 mL of solution. 15.0 mL of a 0.50 mol/L silver nitrate solution was required to precipitate all the chloride ions from this solution. The two possible precipitation reactions are

   $NaCl(aq) + AgNO_3(aq) \rightarrow AgCl(s) + NaNO_3(aq)$

   $CaCl_2(aq) + 2\ AgNO_3(aq) \rightarrow 2\ AgCl(s) + Ca(NO_3)_2(aq)$

   Was the unknown solute calcium chloride or sodium chloride? T/I
8. Calculate the amount concentration of the cations in the following solutions: T/I
   (a) 0.5 mol/L sodium carbonate, Na_2CO_3
   (b) 0.2 mol/L ammonium sulfite, $(NH_4)_2SO_3$
   (c) 1.5 mol/L iron(III) sulfate, $Fe_2(SO_4)_3$
9. What mass of sodium carbonate, $Na_2CO_3(s)$, is required to prepare 200.0 mL of a solution in which the sodium concentration is 0.85 mol/L? T/I